Title: Patterns, Patterns, Everywhere!

Brief Overview:

> The students will write and analyze equations of certain functions from generating tables, observing patterns, and using regression analysis on the TI-83 calculator. They will find the sum of the interior angles of a convex polygon, discover the number of distinct handshakes given a set number of people, and determine the number of shaded triangles at certain stages of the Sierpinski Gasket.

Links to NCTM 2000 Standards:

- **Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation**
  These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

  Students will be able to complete charts, develop patterns, and complete regression analysis on data. They will be able to explain the relationship between data points and regression equations. They also will be required to write about patterns generated from data and to justify their conclusions. Students will investigate ways to solve a complex problem by simplifying the problem, drawing conclusions from patterns, and applying algebra with geometry. Last of all, they will demonstrate knowledge of representing data in multiple forms. These forms include data lists, equations, and graphs.

- **Number and Operation**
  Students will understand ways of representing numbers and relationships among numbers by using computational tools and strategies fluently and estimating appropriately.

- **Patterns, Functions, and Algebra**
  Students will understand various types of patterns and functional relationships by using mathematical models and analyzing change in both real and abstract contexts.

- **Geometry and Spatial Sense**
  Students will analyze characteristics and properties of two-dimensional geometric objects.

- **Data Analysis, Statistics, and Probability**
  Students will interpret data using methods of exploratory data analysis and develop and evaluate inferences, predictions, and arguments that are based on data.

Links to Virginia High School Mathematics Core Learning Units:

- **Geometry (G.14)**
  Students (given similar geometric objects) will use proportional reasoning to solve practical problems. They will investigate relationships between linear and quadratic functions.

- **Algebra II (AII.2)**
  Students will multiply, divide, and simplify rational expressions.

- **Algebra II (AII.7)**
  Students will solve equations containing rational expressions algebraically and graphically. Graphing calculators will be used for solving and confirming algebraic solutions.

- **Algebra II (AII.8)**
  Students will recognize multiple representations of functions and convert between a table, a graph, and symbolic form. The students will use a transformational approach when using graphing calculators.

- **Algebra II (AII.9)**
  Students will find zeros and values of functions given an element in its domain. The graphing calculator will be used as a tool to investigate linear, quadratic, and exponential functions.

- **Algebra II (AII.10)**
  Students will investigate and describe the relationships between the solution of an equation, zero of a function, and factors of a polynomial expression through the use of graphs.

- **Algebra II (AII.19)**
  Students will collect and analyze data to make predictions, write equations, and solve practical problems. The students will use graphing calculators to investigate scatter plots and to determine the equation for a curve of best fit.

Grade/Level:

Grades 9-12; Advanced Geometry, Algebra II, Pre-calculus, Calculus, Discrete Mathematics, and Statistics

Duration/Length:

Two 90-minute blocks or three to four 50 minute classes

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Applying fractions
- Understand angle measures, triangles and polygons
- Manipulating exponential expressions
- Entering data into lists on the TI-83
- Graphing scatter plots
- Calculating regression analysis on the TI-83

Objectives:

Students will:

- find the sum of the interior angles of any convex polygon.
- discover the number of distinct handshakes given a set number of people.
- determine the number of shaded triangles at certain stages of the Sierpinski Gasket.
- work cooperatively.
- write and analyze equations of certain functions from generating tables, observing patterns, and using regression analysis on the TI-83 calculator.

Materials/Resources/Printed Materials:

- Triangle paper from *Making a Fractal: The Sierpinski Triangle (by Cynthia Lanius)*
  http://math .rice. edu/~lanius/fractals/
- TI-83 graphing calculators
- Straight edges (optional)

Development/Procedures:

We recommend students work in groups of two or three, so that they can discuss the topics and help each other formalize the patterns and procedures as they utilize the graphing calculator.   These activities are primarily student-based discovery lessons. However, it is essential to regroup as a class to make sure that the main ideas are being developed.

It is not necessary that all three activities be completed as a unit, or all in consecutive order. The first activity explores linear regression, the second activity compares linear and quadratic regression, and the final activity investigates exponential patterns and regression. The teacher may select the activity that is appropriate for the students. After the class has finished each activity, it is important that the teacher lead discussions of the main concepts.

Assessment:

In evaluating these activities the teacher should consider how well the students work cooperatively in class. The teacher will circulate about the classroom as the groups work, ensure the students stay on task, and provide hints as needed.

Completion of the worksheet and discussion will be evaluated. In each activity students should demonstrate an understanding of the specific mathematical points (e.g. definition of convex polygon, sum of interior angles, relationships between diagrams and patterns). Also, students should develop correct patterns with symbolic and graphical representations.

Extension/Follow Up:

- After completing Activity 3 (Sierpinski Gasket), teachers can discuss fractals and their paradoxical nature.
- Students can generate their own fractals and determine the patterns that exist within the ones they create.
- Students can also research fractals in nature.
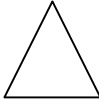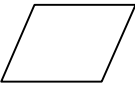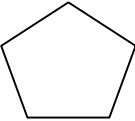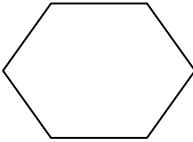
Authors:
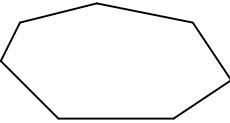
Ellen Y. Chien   Rita M. Driskill
Langley High School   Broad Run High School
Fairfax County, VA   Loudoun County, VA

Student Activity 1: Polygon Angle Sums   **Name**

_____
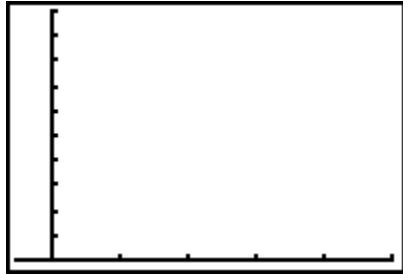
1.   Draw a convex polygon.
        Classify the polygon by the number of sides it has. ____
        Explain why is it convex. _____

2.   Draw a triangle.
        From *one vertex*, how many diagonals can you draw? ____
        How many triangles are formed? _____
        What is the sum of the measures of the interior angles in this polygon? _____

3.   Draw a quadrilateral.
        From *one vertex*, how many diagonals can you draw? _____
        How many triangles are formed? _____
        What is the sum of the measures of the interior angles in this polygon? _____

4.   Complete the table below and see if you can write equations generalizing the patterns at the end
        of each column. Draw diagrams and discuss your results with your group.

Diagram	# of Sides = L_1	Name of Polygon	# of Diagonals from One Vertex	# of Non-Overlapping Triangles	Sum of Interior Angles = L_2
	3				
	4				
	5				
	6				
	7				
No Picture available	n	n-gon			

Student Activity 1: Polygon Angle Sums   **Name**

_____

5.   Using your calculator, enter **# of Sides** into L_1 and the **Sum of the Interior Angles** in L_2.
        Turn STATPLOT on and graph L_1, L_2 with the data plot line.
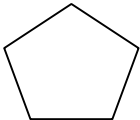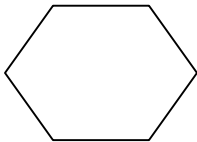   *Set your window to [-1,10]$_2$ ´ [-1, 1000]$_{100.}$*
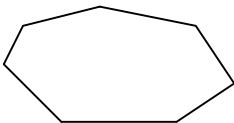

        Does the pattern appear to be linear in nature? Explain.

6.   Use the TI-83 regression analysis to determine the equation for the sum of the interior angles of
        an n-gon. Which regression did you use? Explain.

        y = _____   r = _____

        What does r, the correlation coefficient, tell you about how well the equation
   predicts the pattern? Explain.

Answer Key and Teacher Notes to Student Activity 1: Polygon Angle Sums

1.   Draw a convex polygon.
     Classify the polygon by the number of sides it has.
     Explain why is it convex. *Answers will vary. A convex polygon is a closed figure made of
                                  segments that only intersect at vertices. Match definition to your
                                  textbook.*

2.   Draw a triangle.
     From *one vertex*, how many diagonals can you draw? **0**
     How many triangles are formed? **1**
     What is the sum of the measures of the interior angles in this polygon? **180°**

3.   Draw a quadrilateral.
     From *one vertex*, how many diagonals can you draw? **1**
     How many triangles are formed? **2**
     What is the sum of the measures of the interior angles in this polygon? **360°**

4.   Complete the table below and see if you can write equations generalizing the patterns at the end
     of each column. Draw diagrams and discuss your results with your group.

Diagram	# of Sides = L_1	Name of Polygon	# of Diagonals from One Vertex	# of Non-Overlapping Triangles	Sum of Interior Angles = L_2
	3	Triangle	0	1	180°
	4	Quadrilateral	1	2	360°
	5	Pentagon	2	3	540°
	6	Hexagon	3	4	720°
	7	Heptagon	4	5	900°
No picture available	n	n-gon	n-3	n-2	(n-2)180°

Answer Key and Teacher Notes to Student Activity 1: Polygon Angle Sums

5.   Using your calculator, enter **# of Sides** into L_1 and the **Sum of the Interior Angles** in L_2.
        Turn STATPLOT on and graph L_1, L_2 with the data plot line.
        Set your window to [-1,10]₂ ´ [-1, 1000]₁₀₀.
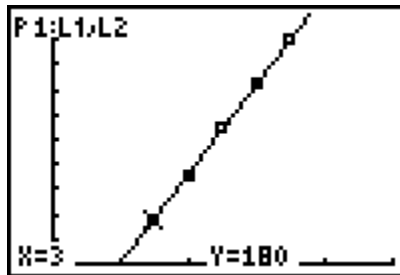
        Does the pattern appear to be linear in nature? Explain.
        * *Yes, because all the data points appear to fall on the same line.*

6.   Use the TI-83 regression analysis to determine the equation for the sum of the interior angles of
        an n-gon. Which regression did you use? Explain.

        * *y = 180x - 360   r = 1*   * *Use* **LinReg** *because data appear to be linear.*

        What does r, the correlation coefficient, tell you about how well the equation
        predicts the pattern? Explain.

        * *Because |r| = 1, the linear equation has a perfect correlation to the data points.*
                In other words, all the plot points lie exactly on the line.
        * *Use* **2ⁿᵈ CALC** *or* **2ⁿᵈ TABLE** *or* **TRACE** *to verify that the data points*
                match the points on the line.

Student Activity 2: How Many Handshakes?   **Name** _____

How Many Handshakes?

1.   If there are 26 people in a room, in your estimate, how many distinct handshakes
      will occur if each person shakes hands? _____

2.   Let's simplify this problem, beginning with no one in the room. If there is no one
      in the room, then no handshakes occur. Continue and complete the table below:

| $L_1 =$ # of People	$L_2 =$ # of Distinct Handshakes
0	0
1	
2	
3	
4	
5	

Hint: You can either simulate the handshakes in small groups or draw polygons
where vertices represent people, then look for patterns in your results and
complete the rest of the table.

# of People	# of Distinct Handshakes
6	
7	
8	

3.   Explain in complete sentences how you got the values in the handshake column.

4.   a) Next, plot the **# of People** versus
          the **# of Distinct Handshakes**.
          Assign the # of People as
          the independent variable.
          Be sure to label your axes!
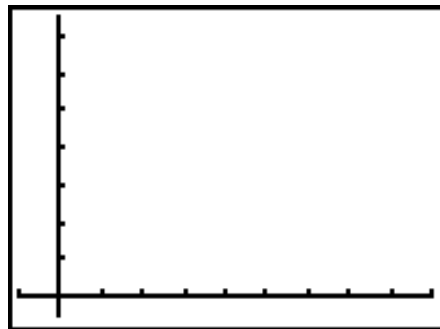
          Set your window to [-1,9]₁ ´ [-2,30]₄.

      b) Does the pattern appear linear in nature? Explain.

5.   a) Find the median-median line and graph it with #4.
                What is the equation of the med-med line?

        y = _____

        b) Now do you think the plot is linear in nature?
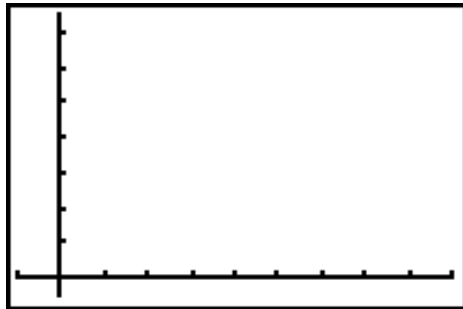   Use the graph to explain.

6.   Try the QuadReg on L_1 and L_2.
        a) State the equation and r value. Hint: Turn DIAGNOSTICS ON.

        y = _____ r^2 = _____   r = _____

        What does r, the correlation coefficient, tell you about the quadratic regression
        equation? Explain.

        b) Graph the data and the quadratic equation on the same axes.

        c) Compare your values from #2 and the table from the TI-83 (2nd TABLE).
   What did you find?

7.   *In conclusion*, how many distinct handshakes will occur in a room with 26
        people?_____ Was your educated guess close? _____

Student Activity 2: How Many Handshakes?   **Name** _____

Extension:

1.   Is the relation between the number of people and handshakes a function? Explain.

2.   a) If the roots of a quadratic equation are $x = 4$ and $x = -2$, then one of possible
          factored equation for these roots is: $(x - 4)(x + 2) = 0$. What are the roots of
          the handshake equation?

       b) Write an equation for your handshake data in factored form.

       c) Plot your answer from b) against your scatterplot of L_1 and L_2 on the graph
           below. Do they match exactly?_____ Why or why not?
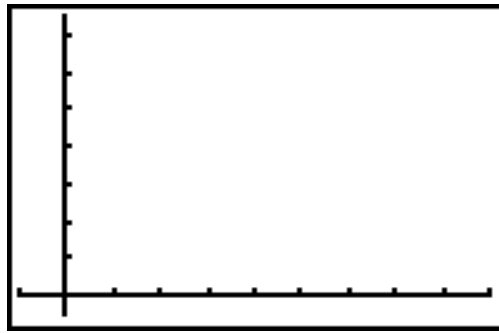
       d) What adjustments can you make to the equation to match your data points
           exactly?

Answer Key and Teacher Notes to Student Activity 2: How Many Handshakes?

How Many Handshakes?

1.   If there are 26 people in a room, in your estimate, how many distinct handshakes will occur if each person shakes hands? *** Answers will vary***.

2.   Let's simplify this problem, beginning with no one in the room. If there is no one in the room, then no handshakes occur. Continue and complete the table below:

$L_1 =$ # of People	$L_2 =$ # of Distinct Handshakes
0	0
1	0
2	1
3	3
4	6
5	10

Hint: You can either simulate the handshakes in small groups or draw polygons where vertices represent people, then look for patterns in your results and complete the rest of the table.

# of People	# of Distinct Handshakes
6	15
7	21
8	28

3.   Explain in complete sentences how you got the values in the handshake column.

* Answers will vary. Students may notice the pattern of :

# of handshakes = previous # of handshakes + the next consecutive integer
beginning with 0

> *ex.   For 1 person, 0 handshakes = 0 + 0.*
> *For 2 people, 1 handshake = 0 + 1.*
> *For 3 people, 3 handshakes = 1 + 2.*
> *For 4 people, 6 handshakes = 3 + 3.*

4.   a) Next, plot the # of People versus the # of Distinct Handshakes. Assign the number of people as the independent variable. Be sure to label your axes!
       Set your window to $[-1,9]_1$ ´ $[-2,30]_4$.
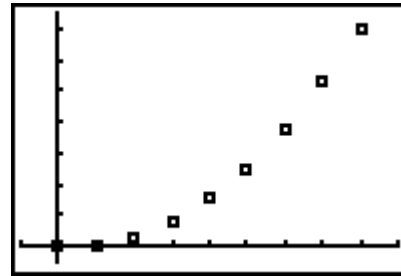
```
WINDOW
 Xmin=-1
 Xmax=9
 Xscl=1
 Ymin=-2
 Ymax=30
 Yscl=4
 Xres=1
```

Answer Key and Teacher Notes to Student Activity 2: How Many Handshakes?

b) Does the pattern appear linear in nature? Explain.

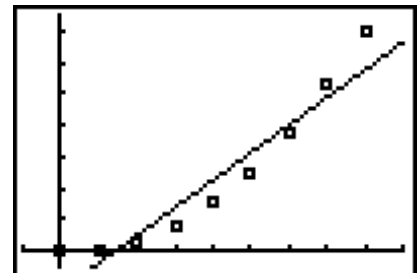** Answers may vary.*

# of Distinct
Handshakes

# of People

5.   a) Find the median-median line and graph it with #4.
          What is the equation of the med-med line?

```
Med-Med L₁,L₂,Y₁
```

```
Med-Med
  y=ax+b
  a=3.5
  b=-5
```

TI-83 Calculator Key Commands for Med-Med Line of Best Fit:
        ** Type **STAT CALC 3:Med-Med L₁ , L₂ , Y₁** on the homescreen.*
        Y₁ *is obtained by typing* **VARS Y-VARS 1:Function 1:Y₁.**

        * y = 3.5x - 5

NOTE: We chose to use the med-med fit line for the data for students to review
        for Virginia's Standards of Learning Tests. However, you may choose to
        perform the linear regression **LinReg** *on the number of people versus the*
        number of distinct handshakes resulting in y = 3.5x - 4.667 with
        r = 0.9514. The students could use the value of r to justify whether the
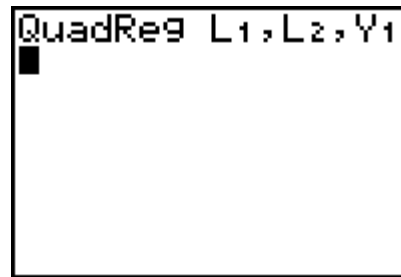   *data is linear.*

b) Now do you think the plot is linear in nature?
        Use the graph to explain.

* Answers may vary.

Answer Key and Teacher Notes to Student Activity 2: How Many Handshakes?

6. Try the QuadReg on L_1 and L_2.
   a) State the equation and r value.
       Hint: Turn DIAGNOSTICS ON.

    ```
    QuadReg L₁,L₂,Y₁
    ■
    ```

    $* y = 0.5x^2 - 0.5x \qquad r^2 = 1 \qquad r = 1$

    What does r, the correlation coefficient, tell you about the quadratic regression equation? Explain.

    > *Because ǀrǀ = 1, the quadratic equation has a perfect correlation to the data points. In other words, all the plot points lie exactly on the curve.*

    b) Graph the data and the quadratic equation on the same axes.
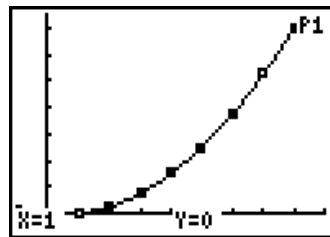
    c) Compare your values from #2 and the table from the TI-83 (2nd TABLE).
       What did you find?

    > *The values from each table should match exactly demonstrating the quadratic equation fits the data exactly.*

    ```
    X    Y₁
    0    0
    1    -4E-13
    2    1
    3    3
    4    6
    5    10
    6    15
    X=0
    ```

    > *-4Ē13 is equal to zero due to the calculator's rounding. Have students verify that x=1 yields y=0 from the y = equation.*

7. In conclusion, how many distinct handshakes will occur in a room with 26 people?

    > * There will be 325 handshakes.
    >   Use 2nd CALC or 2nd TABLE or TRACE to find
    >   the y-value when x is 26.*

Answer Key and Teacher Notes to Student Activity 2: How Many Handshakes?

Extension:

1.   Is the relation between the number of people and handshakes a function? Explain.

>   * *This relation is a function.   The domain does not repeat.*
>   *Also, it passes the vertical line test.*

2.   a) If the roots of a quadratic equation are x = 4 and x = -2, then *a* factored
>   equation for these roots is: (x - 4)(x + 2) = 0. What are the roots of the
>   handshake equation?

>   * *The roots are x = 0 and x = 1.*

>   b) Write an equation for your handshake data in factored form.

>   * *Answers may vary. However, most students will give x(x - 1) = 0.*

>   c) Plot your answer from b) against your scatterplot of L₁ and L₂ on the graph
>   below. Why don't they match exactly?
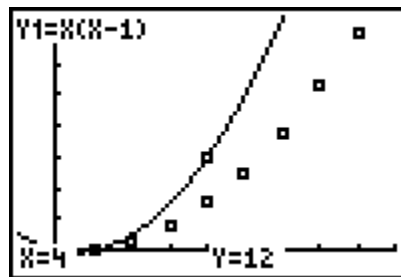
>   * *The graph of y = x(x - 1) is too narrow.*
>   *It does not pass through many of the data points.*

>   d) What adjustments can you make to the equation to match your data points
>   exactly?

>   * *Answers may vary. The graph needs to be made wider.*
>   *From transformations, we must multiply our equation by a number*
>   *between 0 and 1. Hopefully, the students will try 0.5 as the factor, and*
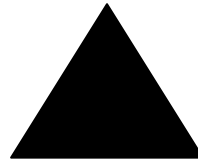>   *verify with the quadratic regression equation.*

Student Activity 3: Introduction to the Sierpinski Gasket   Name _____

Egads! **It's the Sierpinski Gasket!**

The Sierpinski Gasket (or Triangle) is one of the most famous fractals studied in mathematics. You can easily create one using any type of triangle. You can either use the *triangle paper* provided or a straight edge to create a three stage Sierpinski Gasket.

Stage Zero

Using the triangles on the worksheet, begin by drawing a triangle whose sides are easily divisible by 2. Shade this triangle lightly with a pencil.

Stage One

Measure and locate the midpoints of each side. Then connect the midpoints.
How many triangles are there? _____
Leave the center triangle unshaded as shown.

Stage Two

Using the previous triangle, connect the midpoints of the sides of the remaining black triangles, and leave the center of each of the smaller triangles unshaded as shown.

Stage Three

Repeat Stage Two, bisecting the sides and connecting the midpoints of the remaining black triangles, and leaving each of the smaller center triangles unshaded.
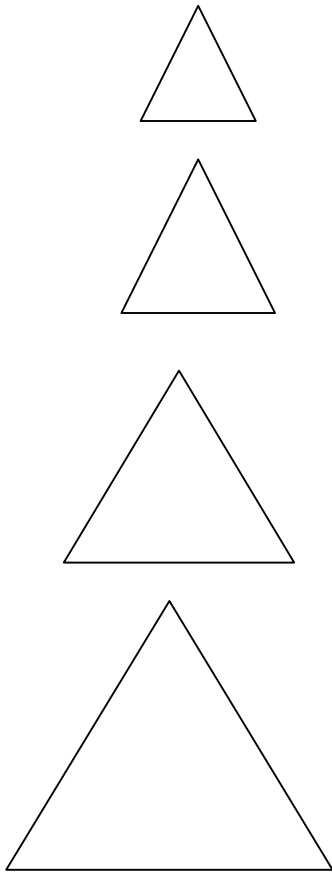

Stage Infinity

This iteration can be done ad infinitum. Follow the above pattern and complete the Gasket. Be as creative and as colorful as you like!
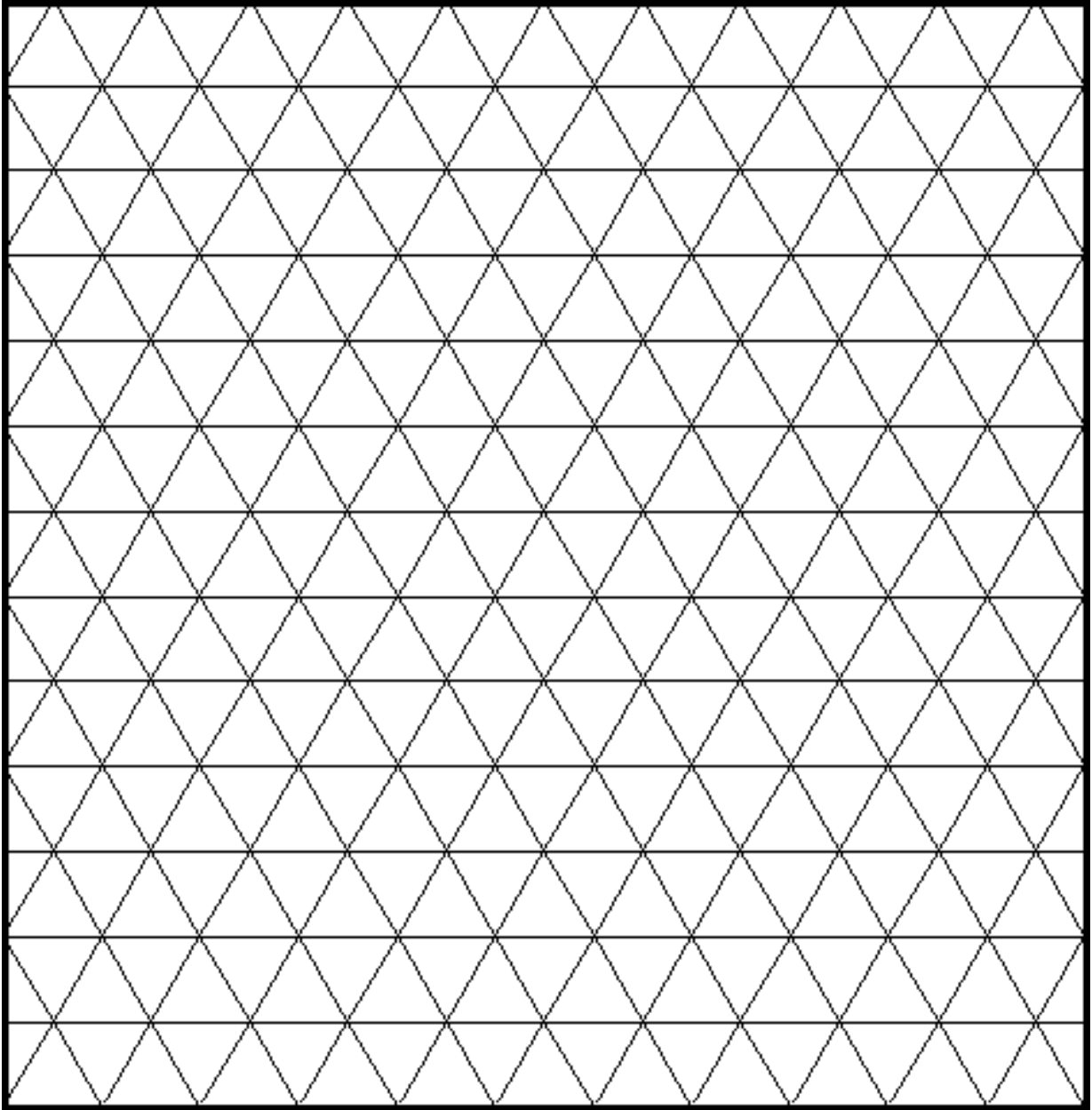

Images of Sierpinski Triangles from Making a Fractal: The Sierpinski Triangle (by Cynthia Lanius)
http://math.rice.edu/~lanius/fractals

Student Activity 3

PATTERNS WITH SIERPINSKI

Stage Number	# of Shaded Triangles	Area of One Shaded Triangle	Area of Shaded Region(s)	Length of One Side of a Shaded Triangle	Perimeter of One Shaded Triangle	Perimeter of All Shaded Triangles
Stage Zero	1	1	1*	1	3	3
Stage One	3	¼	¾	½	3/2	9/2
Stage Two	9					
Stage Three						
Stage x						

* Set area of original triangle in Stage Zero as 1 unit^2.

Taken from

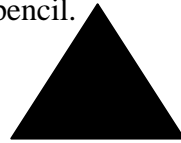*Making a Fractal: The Sierpinski Triangle (by Cynthia Lanius).* http://math.ric.edu/~lanius/fractals

Teacher Notes to Student Activity 3: Introduction to the Sierpinski Gasket

Egads! **It's the Sierpinski Gasket!**

The Sierpinski Gasket (or Triangle) is one of the most famous fractals studied in mathematics. You can easily create one using any type of triangle. You can either use the *triangle paper* provided or a straight edge to create a three stage Sierpinski Gasket.

Stage Zero
Using the triangles on the worksheet, begin by drawing a triangle whose sides are easily divisible by 2. Shade this triangle lightly with a pencil.

Stage One
Measure and locate the midpoints of each side. Then connect the midpoints.
How many triangles are there? *4*
Leave the center triangle unshaded as shown.

Stage Two
Using the previous triangle, connect the midpoints of the sides of the remaining black triangles, and leave the center of each of the smaller triangles unshaded as shown.

Stage Three
Repeat Stage Two, bisecting the sides and connecting the midpoints of the remaining black triangles, and leaving each of the smaller center triangles unshaded.
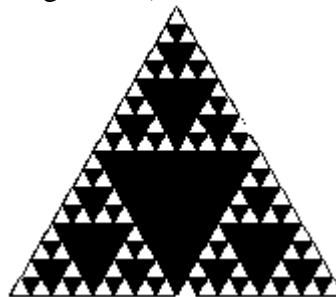
Stage Infinity
This iteration can be done ad infinitum. Follow the above pattern and complete the Gasket. Be as creative and as colorful as you like!
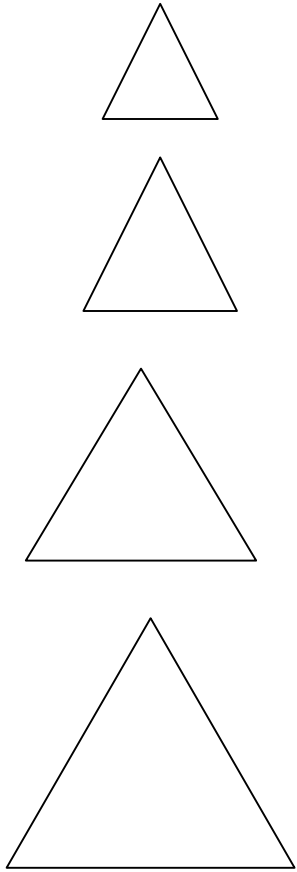
Stage Four   Stage Five *(inverted coloring)*

Images of Sierpinski Triangles from Making a Fractal: The Sierpinski Triangle (by Cynthia Lanius)
http://math.rice.edu/~lanius/fractals

Answer Key and Teacher Notes to Activity 3: PATTERNS WITH SIERPINSKI

Egads! for examples of completed triangles up to Stage Five.

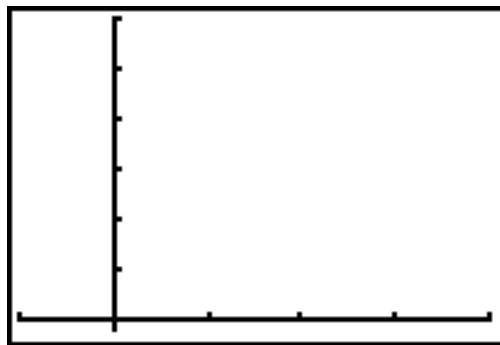| Stage Number	# of Shaded Triangles	Area of One Shaded Triangle	Area of Shaded Region(s)	Length of One Side of a Shaded Triangle	Perimeter of One Shaded Triangle	Perimeter of All Shaded Triangles
Stage Zero	1	1	1*	1	3	3
Stage One	3	¼	¾	½	3/2	9/2
Stage Two	9	1/16	9/16	¼	3/4	27/4
Stage Three	27	1/64	27/64	1/8	3/8	81/8
Stage x	3^x	$(1/4)^x$	$(3/4)^x$	$(1/2)^x$	$3/2^x =$ $3*(1/2)^x$	$\dfrac{3^{x+1}}{2^x} =$ $3*(3/2)^x$

* Set area of original triangle in Stage Zero as 1 unit^2.

Student Activity 3: The Sierpinski Gasket Questions   Name _____

1.   a) Determine the equation for the number of shaded triangles at stage x from your table.

   b) Use the TI-83 regression analysis to determine the equation for the number of shaded triangles at Stage x from your data. Which regression did you use? Explain.

   y = _____   r = _____

   c) Graph **Stage Number** vs. **Number of Shaded Triangles** with the regression equation:
   Set your window to [-1,4]₁ ´ [-1,30]₅.

2.   a) Graph **Stage Number** vs. **Area of One Shaded Triangle** below:
   Set your window to [-1,4]₁ ´ [-1,2]₀.₅.

   What will the area of one shaded triangle of the Sierpinski Gasket approach as the number of stages increases? Explain.

   b) Perform a regression analysis on **Stage Number** vs. **Area of One Shaded Triangle**, and draw the regression equation on the graph above. Which regression did you use? Explain.

   y = _____   r = _____

3.   a) Graph **Stage Number** vs. **Length of One Side of a Shaded Triangle**
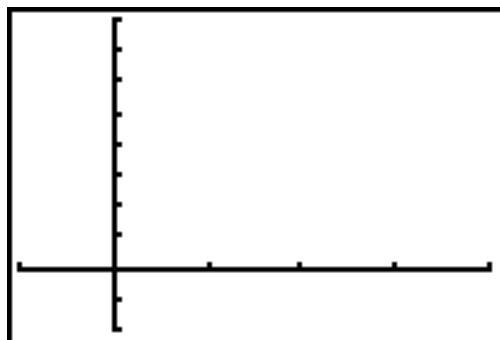   below: *Set your window to [-1,4]$_1$ ´ [-1,1.5]$_{0.5}$.*

What will the length of one side of a shaded triangle of the Sierpinski Gasket approach as the number of stages increases? Explain.

b) Perform a regression analysis **Stage Number** vs. **Length of One Side of a Shaded Triangle**, and draw the regression equation on the graph above. Which regression did you use? Explain.

y = _____ r = _____

4.   a) Graph **Stage Number** vs. **Perimeter of One Shaded Triangle** below:
        Set your window to [-1,4]$_1$ ´ [-1,4]$_{0.5}$.

What will the perimeter of one shaded triangle of the Sierpinski Gasket approach as the number of stages increases? Explain.
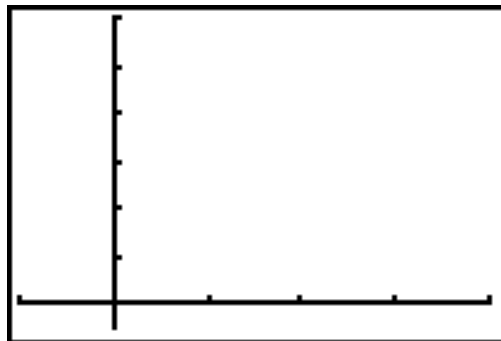
b) Perform a regression analysis on **Stage Number** vs. **Perimeter of One Shaded Triangle**, draw it on the graph above. Which regression did you use? Explain.

y = _____ r = _____

Student Activity 3: The Sierpinski Gasket Questions   **Name** _____

5.   a) Graph **Stage Number** vs. **Perimeter of All Shaded Triangles** below:
         Set your window to [-1,4]₁ ´ [-1,12]₂.

What will the perimeter of all shaded triangles of the Sierpinski Gasket approach as the number of stages increases? Explain.

     b) Perform a regression analysis on **Stage Number** vs. **Perimeter of All Shaded Triangles**, plot on the graph above. Which regression did you use? Explain.

         y = _____ r = _____

Conclusions:

The area of all of the shaded triangles of a Sierpinski Gasket approaches _____ as the number of stages increases towards infinity.

The perimeter of all the shaded triangles of a Sierpinski Gasket approaches _____ as the number of stages increases towards infinity.

Answer Key and Teacher Notes to Student Activity 3:
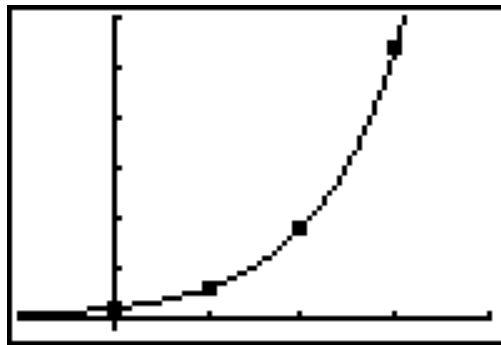**The Sierpinski Gasket Questions**

1. a) Determine the equation for the number of shaded triangles at stage x from your
       table. $* y = 3^x$

    b) Use the TI-83 regression analysis to determine the equation for the number of
       shaded triangles at Stage x from your data. Which regression did you use?
       Explain.

       $* y = 1(3^x)$ $r = 1$ Use **STAT CALC 0:EXPReg** *with stage #*
                                        versus # of shaded triangles because (0,1) was a
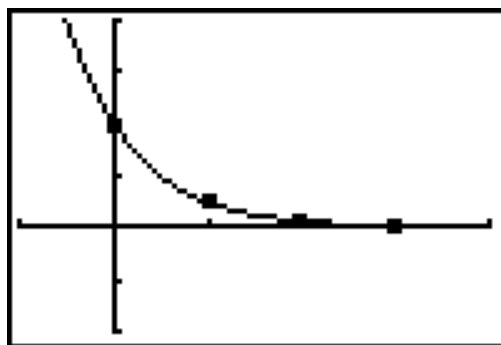   *data point and the graph increased quickly.*

    c) Graph **Stage Number** vs. **Number of Shaded Triangles** with the regression
       equation:
       Set your window to [-1,4]₁ ´ [-1,30]₅.

2. a) Graph **Stage Number** vs. **Area of One Shaded Triangle** below:
        Set your window to [-1,4]₁ ´ [-1,2]₀.₅.

       What will the area of one shaded
       triangle of the Sierpinski Gasket
       approach as the number of stages
       increases? Explain.
       * *The graph of* **Stage Number** *vs.*
         Area of One Shaded Triangle *will*
         approach 0. The graph decreases
         from left to right and does not pass
         through 0.

    b) Perform a regression analysis on **Stage Number** vs. **Area of One Shaded**
       Triangle, and draw the regression equation on the graph above. Which
       regression did you use? Explain.

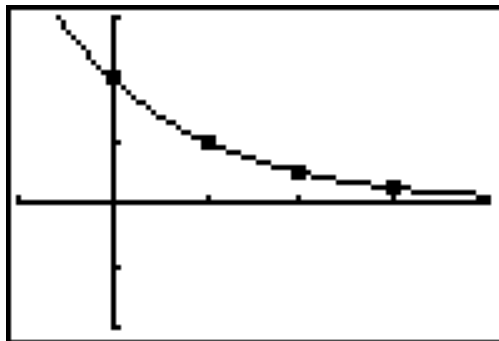 $* y = 0.25^x$ $r = -1$ Use **STAT CALC 0:EXPReg** **Stage Number** vs.
                                        Area of One Shaded Triangle *because (0,1)*
                                        was a data point and the graph decreased quickly.

Answer Key and Teacher Notes to Student Activity 3:
The Sierpinski Gasket Questions

3.   a) Graph **Stage Number** vs. **Length of One Side of a Shaded Triangle**
       below: *Set your window to [-1,4]₁ ´ [-1,1.5]₀.₅.*

What will the length of one side of a
shaded triangle of the Sierpinski
Gasket approach as the number of
stages increases? Explain.
* The graph of the **Stage Number** *vs.*
  Length of One Side of a Shaded
  Triangle *will approach 0. The graph*
  decreases from left to right and does
  not pass through 0.

   b) Perform a regression analysis on **Stage Number** vs. **Length of One Side of a
       Shaded Triangle**, and draw the regression equation on the graph above.
       Which regression did you use? Explain.

       * y = 0.5ˣ   r = -1   *Use* **STAT CALC 0:EXPReg** *with*
                                     Stage Number *vs.* **Length of One Side of**
                                     a Shaded Triangle *because (0,1) was a*
                                     data point and the graph decreased quickly.

4.   a) Graph **Stage Number** vs. **Perimeter of One Shaded Triangle** below:
       Set your window to [-1,4]₁ ´ [-1,4]₀.₅.

What will the perimeter of one
shaded triangle of the Sierpinski
Gasket approach as the number of
stages increases? Explain.
* The graph of the **Stage Number** *vs.*
  Perimeter of One Shaded Triangle
  will approach 0. The graph
  decreases from left to right and
  does not pass through 0.

   b) Perform a regression analysis on **Stage Number** vs. **Perimeter of One
       Shaded Triangle**, draw it on the graph above. Which regression did you use?
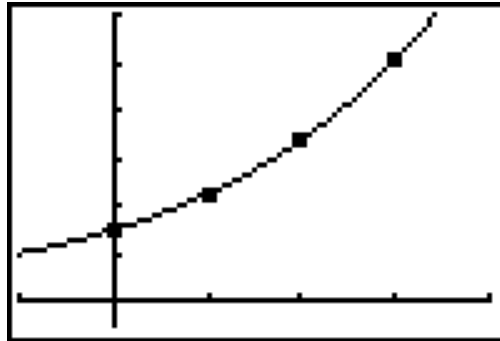   Explain.

       y = 3(0.5)ˣ   r = -1   *Use* **STAT CALC 0:EXPReg** *with stage #*
                                     versus perimeter of one shaded triangle
                                     because (0,1) was a data point and the graph
                                     decreased quickly.

Answer Key and Teacher Notes to Student Activity 3:
The Sierpinski Gasket Questions

5.   a) Graph **Stage Number** vs. **Perimeter of All Shaded Triangles** below:
        Set your window to [-1,4]₁ ´ [-1,12]₂.

What will the perimeter of all shaded triangles of the Sierpinski Gasket approach as the number of stages increases? Explain.
* *The graph of the* **Stage Number** vs. **Perimeter of All Shaded Triangles** *will approach infinity. The graph increases quickly from left to right.*

   b) Perform a regression analysis on **Stage Number** vs. **Perimeter of All Shaded Triangles**, plot on the graph above. Which regression did you use? Explain.

   * $y = 3*(1.5)^x$ $r = 1$ *Use **STAT CALC 0:EXPReg** with **Stage Number** vs. **Perimeter of All Shaded Triangles** because (0,1) was a data point and the graph increased quickly.*

Conclusions:

The area of all of the shaded triangles of a Sierpinski Gasket approaches **_0_**
as the number of stages increases towards infinity.

The perimeter of all the shaded triangles of a Sierpinski Gasket approaches
infinity as the number of stages increases towards infinity.

Extension/Followup:

Generate your own fractal or repeating pattern and determine the formula(s) for the patterns that exist within the ones you create.